Teaching Fraction and Rational Number Concepts with an Area Model Manipulative

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Rationale

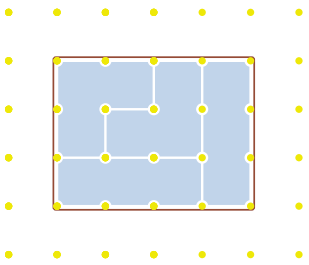
The National Council of Teachers of Mathematics, in its *Principles and Standards for School Mathematics*, describes the need for students of all levels to understand and represent fraction and rational number concepts, recommending that students use a variety of concrete models to support their thinking about abstract ideas and assist them in problem solving.

This area model can be used in a wide range of grade levels for a variety of concepts including introducing fractions, equivalence, comparison of rational numbers, improper fractions, mixed notation, operations, complex fractions, decimals and percent. Fractions with any denominator from 1 to 100 can be modeled.

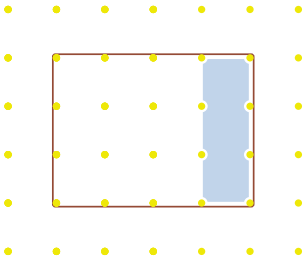
Using the Area Model

The *Fraction Islands*® model uses ten colors of "islands" that are placed on an 11-pin by 11-pin geoboard. All islands of a given color model the same unique area, even though they may have different shapes. Islands cover from one to ten geoboard squares.

To model a fraction, a *unit region* is chosen and bounded by a rubber band. This *unit* represents *one*. The student uses islands of one color to fill the unit, as shown in this figure. *How many fill the unit* determines the *denominator*.

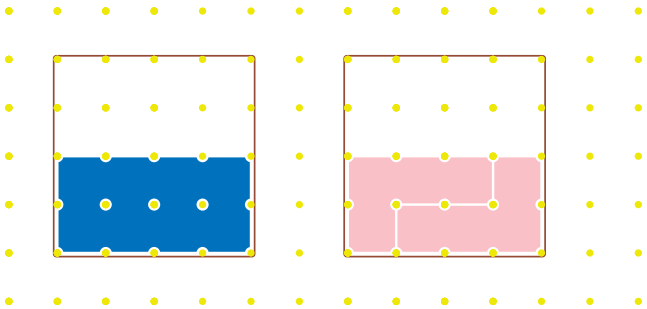


As shown in the next figure, three of the islands have been removed so that only one island is seen in the unit. *How many are seen* determines the *numerator*. Thus, this models ¼.

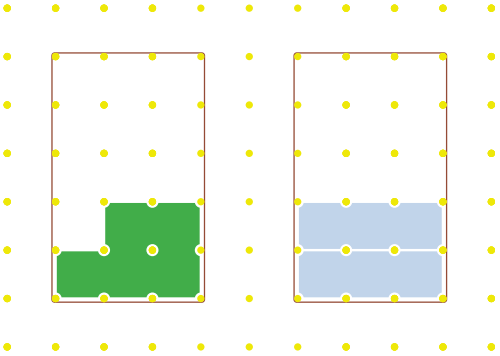


Equivalent Fractions

In this example, the student finds that two dark blue islands fill a chosen unit, then shows only one of the islands in the unit, modeling one-half. In another model of the same unit, the student finds that four pink islands fill the unit. Two of the pink islands, modeling two-fourths, exactly cover the dark blue island, showing that 1/2 is equivalent to 2/4.



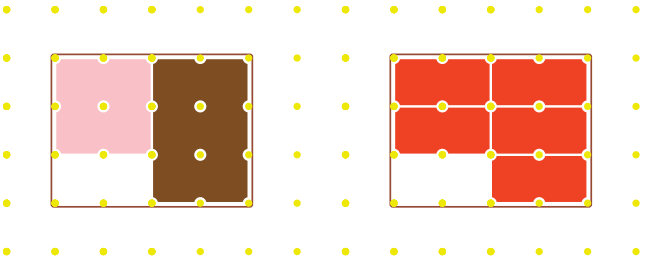
Comparing Rational Numbers



To find which is greater, 1/3 or 2/5, the student models each number in the same size unit, as in this example, then makes a direct comparison by placing the green island that represents 1/3 over the two light blue islands that represent 2/5, to find that the light blue islands occupy more area; thus 2/5 is greater than 1/3.

Addition of Rational Numbers

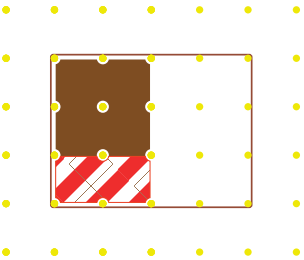
To find the sum of 1/3 and 1/2, the student models both in the same unit, then covers the islands with other islands of a single color that could be used to fill the unit. In this model, a pink island represents 1/3, and a brown island represents 1/2. Red islands may be used to cover both of the islands. The five red islands represent the sum 5/6 in this unit. Another student may cover both islands with yellow islands, each of which covers one geoboard square, showing the sum as 10/12; but 5/6 and 10/12 are equivalent, so represent the same rational number.



Subtraction of Rational Numbers

To find the difference of two rational numbers, the direct comparison method is used, as described above. Then the student can find "2/5 is how much greater than 1/3" by covering the light blue that "sticks out" when the green island is placed over the two blue islands. This can be covered by a yellow island that models 1/15 in this unit.

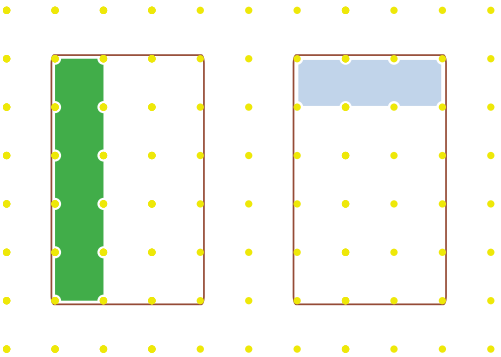
Multiplication of Rational Numbers



Finding the product of 1/3 and 1/2 is modeled as "1/3 of 1/2." 1/2 is modeled with one brown island in this unit. Since three red islands cover the brown island, then one red island models one-third of the brown area. The one red island represents 1/6 in this unit; thus the product is 1/6.

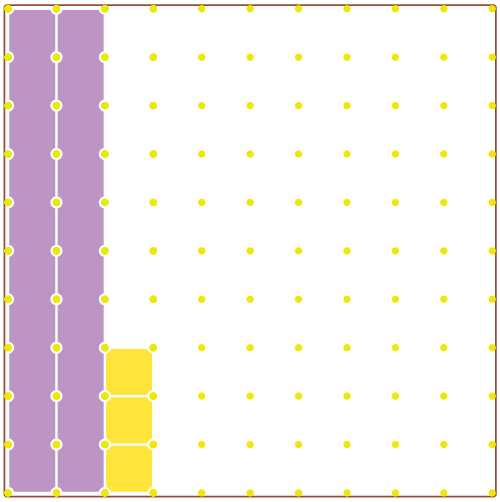
Division of Rational Numbers

For cardinal numbers, "6 divided by 2" can be interpreted as "how many sets of two are the same size as 6." Similarly, in this area model, "1/3 divided by 1/5" is interpreted as "how many sets of 1/5 cover the same area as 1/3." The student models both numbers in the same size unit, then compares to find that more than one set of 1/5, but less than two sets of 1/5 would cover the 1/3. It takes one set and two-thirds of another set of 1/3; So 1/3 divided by 1/5 is 1 2/3.



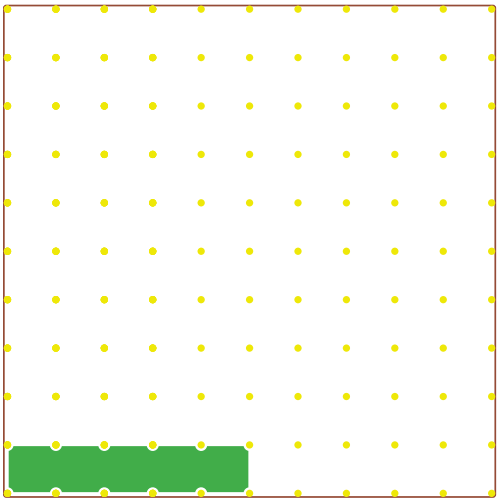
Decimal Notation

The entire geoboard containing 10 rows of 10 squares is used as the unit. Two purple islands model 2/10. Three yellow islands model 3/100. The numeral, 0.23, shows 2 in tenths place and 3 in hundredths place. Using the equivalence concept, the two purple islands may be covered with 20 yellow islands to show 23 yellow islands altogether, thus we name 0.23 "twenty-three hundredths."



Percent

The area model relates percent to both the common fraction notation and the decimal notation. For example, with the entire geoboard as the unit, one green island represents 1/20. The green island can be covered with five yellow islands to model the equivalent fraction, 5/100 or 0.05. We could interpret 5/100 as "5 of 100" or "5 per 100" or "5 percent."



References

National Council of Teachers of Mathematics. *Principles and Standards for School Mathematics*. Reston, Va.:NCTM, 2000.

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